P1 EP1 solutions

## Supervision questions

Please see:

* Why does this not work?

## 1. Rolling race

Under no-slip, the instantaneous centre of a rolling ball is its base. Hence, with speed , angular velocity and radius :

Conserving energy,

When the slope angle satisfies , this suggests that the distance down the slope is . A hollow cylinder has so the final velocity is

Using the property that ,

## 2. Hoop on pegs

As in the previous question, , so applying the parallel axis theorem, . Note that . Applying Newton’s Third Law,

Initially, with , taking moments about the centre of the hoop gives that the clockwise moment generated by the tangential reaction balances ,

Resolving the central weight force in the direction gives the normal reaction

For no-slip, , which occurs when

## 3. Pencil

Conserving angular momentum about ,

The motion of the centre of mass relative to is given by

The -displacement of the tip at collision time is hence

for the forwards velocity to balance the rotation, enabling exact collision at the initial -coordinate. From the previous relationship between and ,

Hence, by calculator (this is a transcendental equation),

### Why does this not work?

The centre of mass is close enough for the tip of the pencil to touch the table when . For ,

So the position of the centre of mass at collision is

Hence,

From here it is unclear how to proceed…

I did try conserving linear momentum horizontally,

And the constant rotation speed is

But still got stuck.

## 4. Horizontal rod and particle

Constrained by , the motion of the particle is

There are no radial forces on the particle, hence

Applying the auxiliary equation , the solution is trivially

For escape at ,

Hence the time of particle escape is

The force acting on the rod acts in the direction and is

Hence, at the moment of escape,

Applying Osborne’s rule,

Then a simplified expression for is

Therefore the anti-clockwise torque at this instant is

## 5. Slider-crank

The instantaneous centre of is at the intersection of the lines through and . Point is a distance from both and the instantaneous centre, hence

So, at the instant shown,

To resist the force , a force of must be applied compressively at along . Hence the torque required at is

must dissipate all of the power transferred into the system by ,

Note that because of the combined opposing motions of and .

## 6. Lifting mechanism

Considering point and an extension of the slider to , the instantaneous centre of the slider is below . Point travels at left horizontally which corresponds to a counter-clockwise rotation speed of about its instantaneous centre. This is also the rotation speed of

The power transferred to the mass is and must be produced at by the input power . Hence

When an additional friction force is applied between the rod and the slider at and a friction torque acts at , this reduces the power transfer by

where is the slider velocity towards . Then,

## 7. Accelerating rod

The acceleration image theorem states that for rigid bodies, the geometric relation of the acceleration vectors is the same as that of the position vectors.

Since the rotating unit vectors instantaneously align with the conventional Cartesian unit vector set,

Comparing to the Databook equations under ,

Then it it trivial to solve for and , under a positive anticlockwise angle convention,

## 8. Sliding block on rod

In the given coordinate system,

When instantaneously,

A fixed point on the rod beneath moves as if it were a stationary , with ,

The acceleration image theorem states that for rigid bodies, the geometric relationship of the acceleration vectors is the same as that of the position vectors. Note that the length is

## 9.

There is no net acceleration since and move at constant velocity.

Considering the component,

Considering the case,

Consider the motion of relative to . Since moves with no acceleration,

It is not possible to determine the direction of the component of .

## 10.

At a general crank angle , by considering the limiting cases of we have

Conserving power transfer through the system in the d’Alembert reference frame,